Time Series Analysis

7th of February 2023

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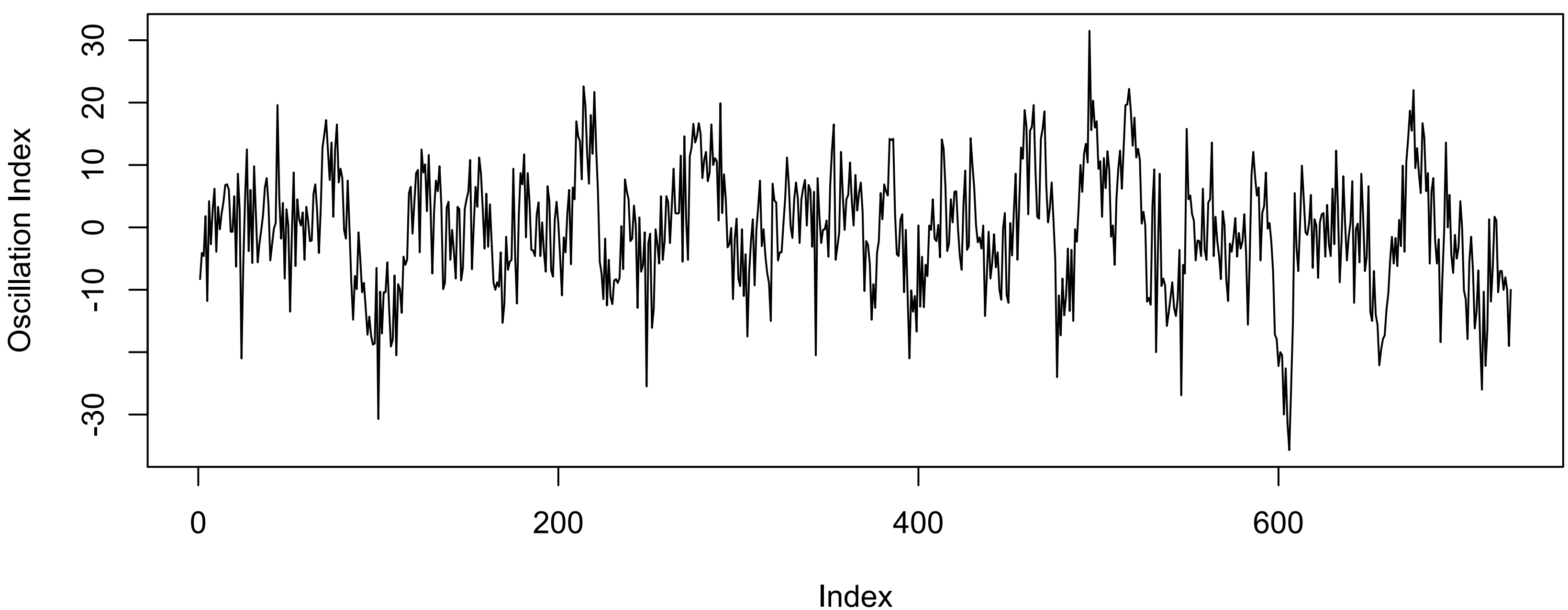
Time series analysis

Example: Monthly Southern Oscillation Index

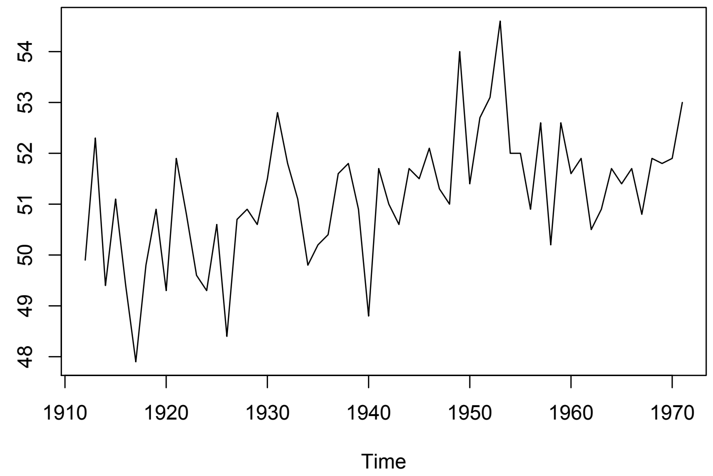
Monthly difference in sea-surface air pressure between Darwin and Tahiti



Used for predicting rainfall in parts of Australia

Time series analysis

A time series is a process in which a given observation depends on other datapoints in the same series.



Linear regression models:
- Response variable (y)
- Independent variables (x)

Time series:
- Single process (y)

Idea:
- Exploit correlations within the data in order to understand and model the data
- Potentially forecast likelihood of future events

Time series analysis

When analysing time series, we are interested in how two values in the series – separated by k time-steps – affect each other.

k^{th} autocovariance:

$$\gamma_k = E(y_t - \mu)(y_{t-k} - \mu)$$

K: Lag

Average covariance between pairs of values that are k time steps apart in the series.

Since these are dependent on the scale of the process, these need to be standardised:

k^{th} autocorrelation:

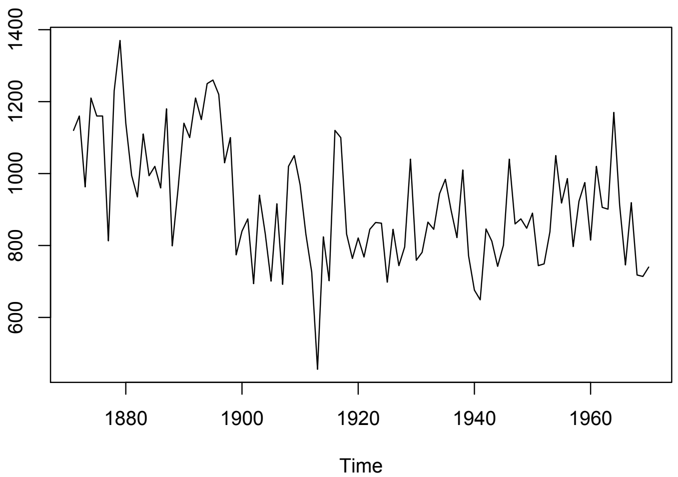
$$\rho_k = \frac{\gamma_k}{\gamma_0}$$

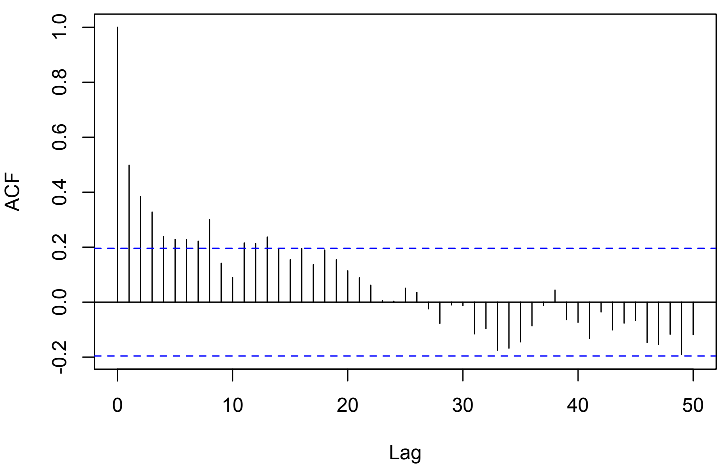
The autocorrelation function is useful for characterising time series.

Time series analysis

Autocorrelation function:

Nile annual flow:





Time series analysis

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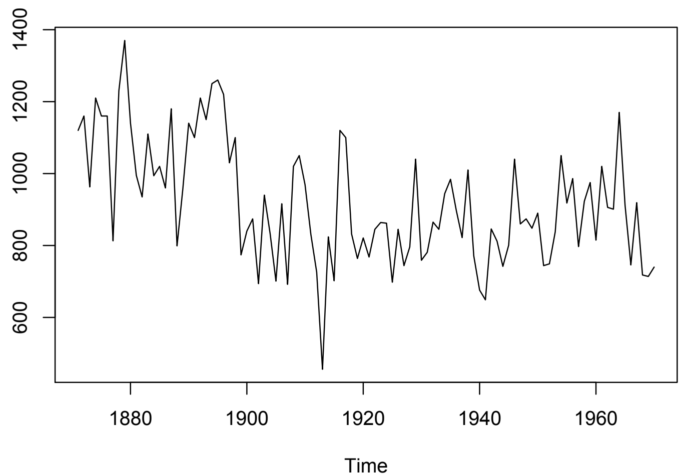
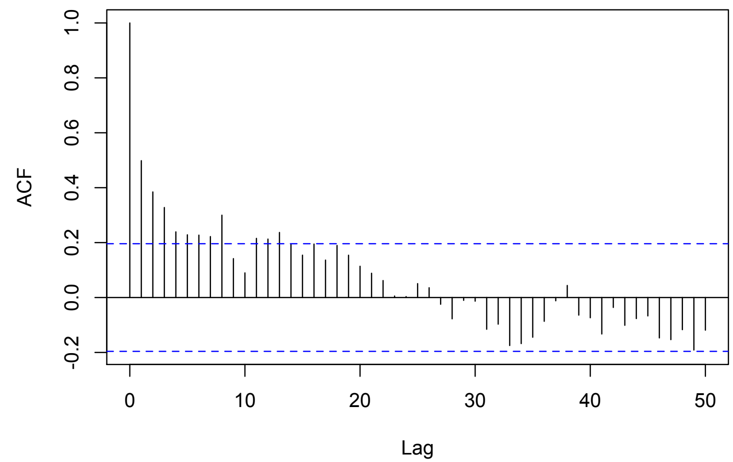
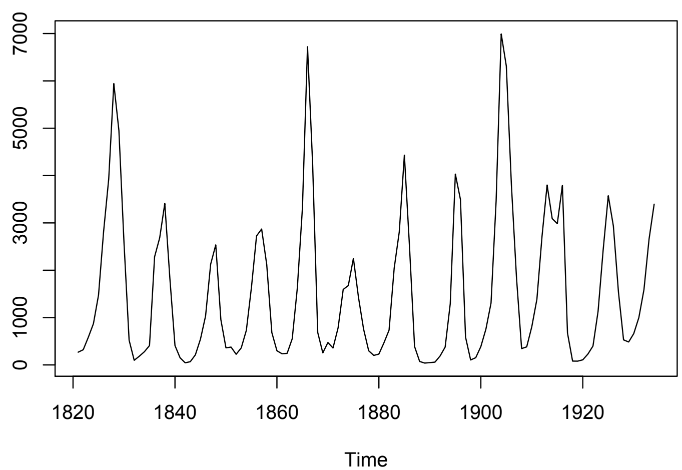
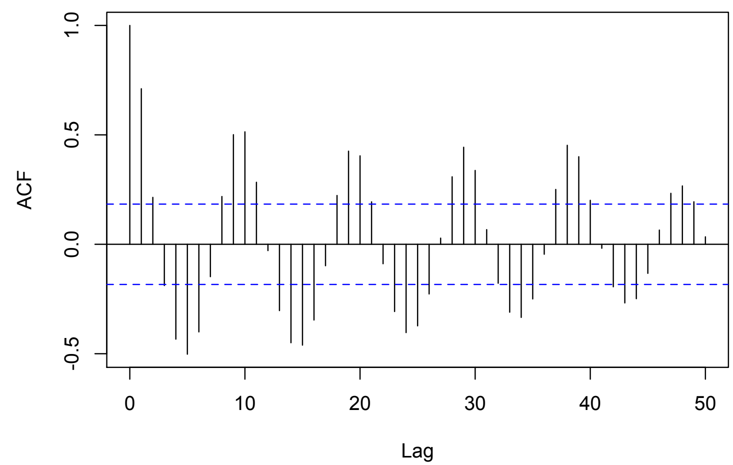




Lynx trappings:





Time series analysis

Autoregressive (AR) time series models:

AR(1): $$y_t = c + \varphi_1 y_{t-1} + \varepsilon_t$$

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AR(p): $$y_t = c + \varphi_1 y_{t-1} + \cdots + \varphi_p y_{t-p} + \varepsilon_t$$

Similarities to multiple regression model, except for the dependencies
Parameters estimated using least squares or maximum likelihood

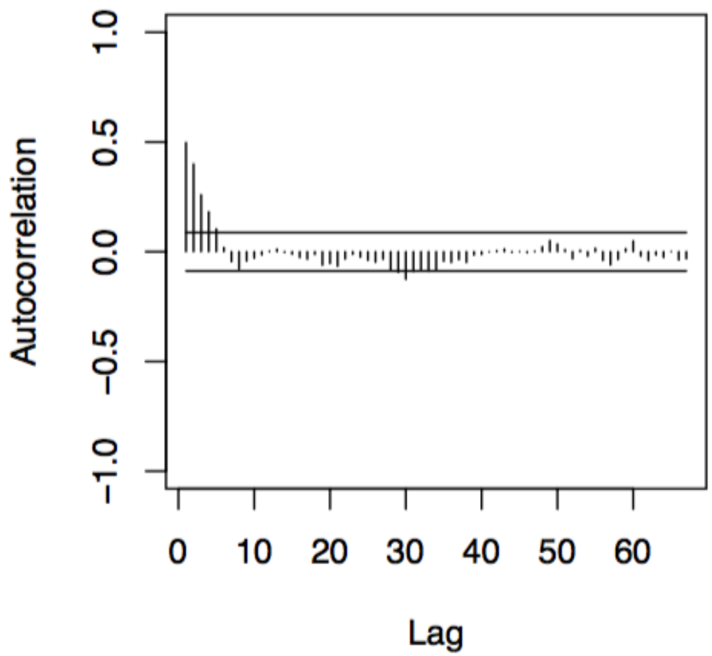
Assumptions:

- Independent Gaussian errors
- Covariance stationary process (trend doesn't change over time)

Time series analysis

Autoregressive (AR) time series models:

AR(2) with $c=0$, $\varphi_1=0.4$ and $\varphi_2=0.2$



How to interpret ACF?

- Positive parameters: ACF should decay, not oscillate.
- Should decay gradually until within the confidence interval, then stay there.
- Can't infer order…

Time series analysis

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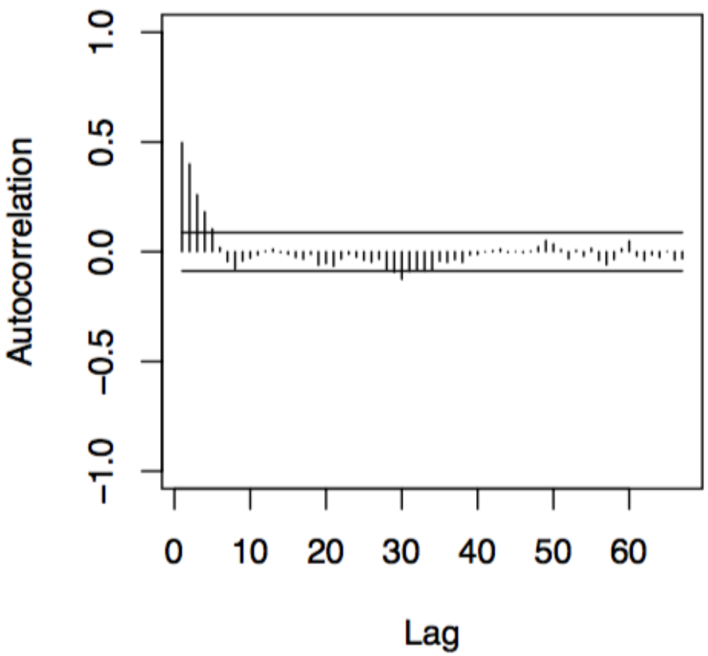

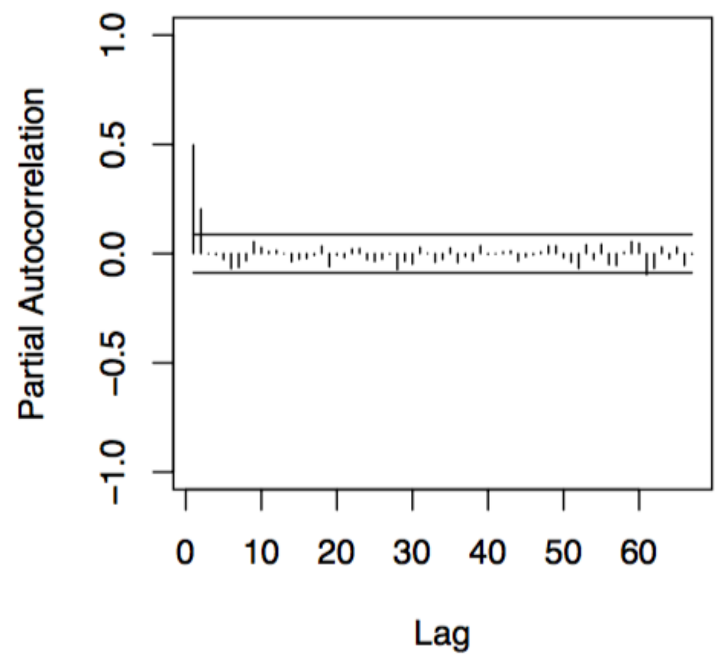

Partial autocorrelation function: $\alpha(p) = \varphi_p$ from a AR(p) model

Parsimonious modelling:

- First try AR(1), then AR(2), etc. until $H_0 : \alpha(p) = 0$ is not rejected.
- Failure to reject leads us to conclude that AR(p) is more appropriate than AR(p-1).

Time series analysis

Moving Average (MA) time series models:

MA(1): $$y_t = c + \varepsilon_t + \theta_1 \varepsilon_{t-1}$$

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Time series analysis

Moving Average (MA) time series models:

MA(1): $y_t = c + \varepsilon_t + \theta_1 \varepsilon_{t-1}$

MA(2): $y_t = c + \varepsilon_t + \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2}$

MA(q): $y_t = c + \varepsilon_t + \theta_1 \varepsilon_{t-1} + \cdots + \theta_q \varepsilon_{t-q}$

Unlike multiple regression model there are multiple error terms
However, the current state is only ever dependent on a known no. of previous states

Since the current state only depends on the previous q states,
the ACF should suddenly drop to zero, unlike AR(p) processes

Time series analysis

More general models:

Auto Regressive, Moving Average:

ARMA(p,q): $$y_t = c + \underbrace{\varphi_1 y_{t-1} + \cdots + \varphi_p y_{t-p}}_{\text{AR(p)}} + \varepsilon_t + \underbrace{\theta_1 \varepsilon_{t-1} + \cdots + \theta_q \varepsilon_{t-q}}_{\text{MA(q)}}$$

Time series analysis

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Auto Regressive, Integrated, Moving Average:

ARIMA(p,1,q): $x_t = y_t - y_{t-1}$ then model as ARMA(p,q)

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ARIMA(p,1,q): $x_t = y_t - y_{t-1}$ then model as ARMA(p,q)

ARIMA(p,d,q): $x_t = \nabla^d y_t$ take d^{th} order differences

Considering ARIMA models can be a useful "transformation" if assumptions are violated

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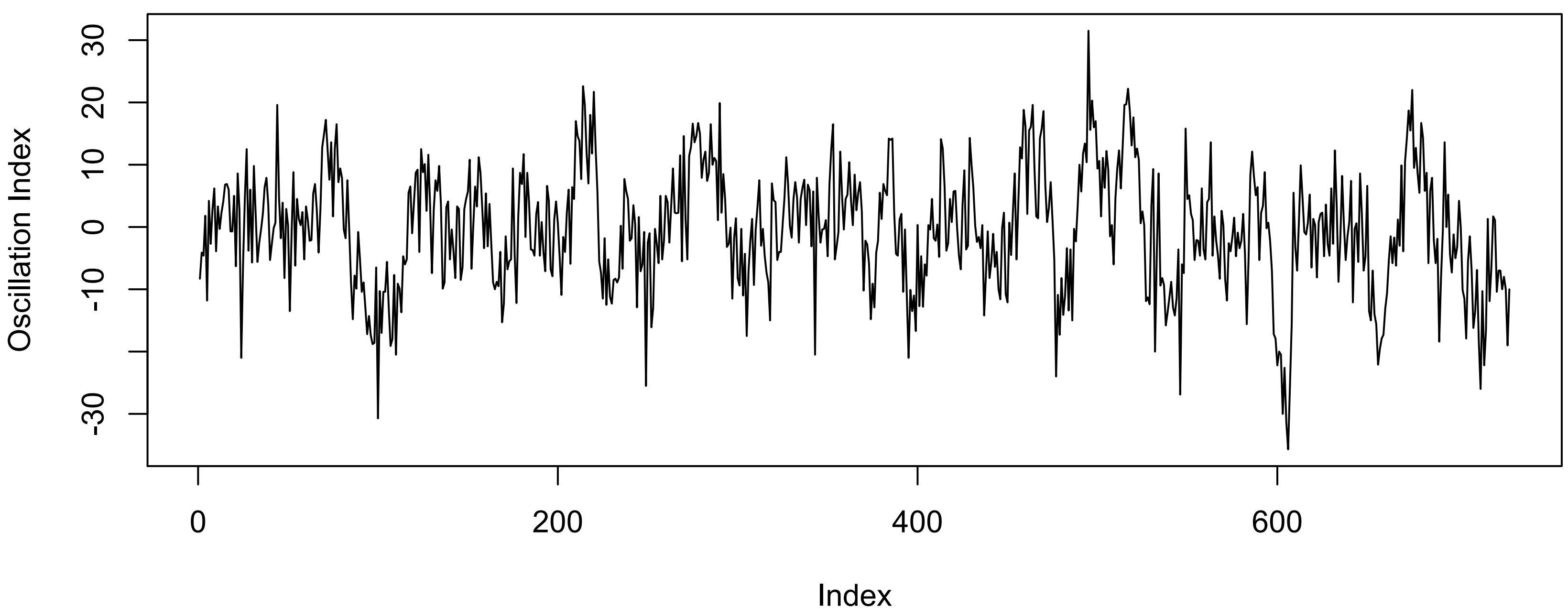

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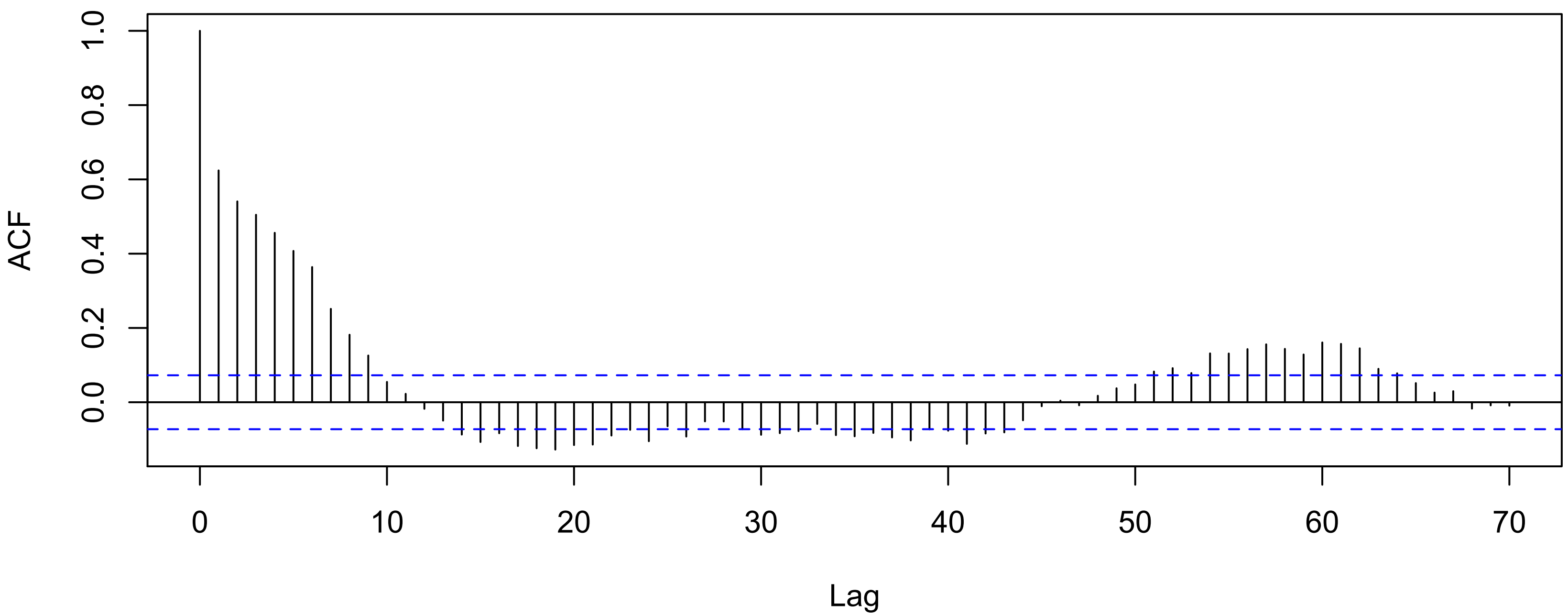

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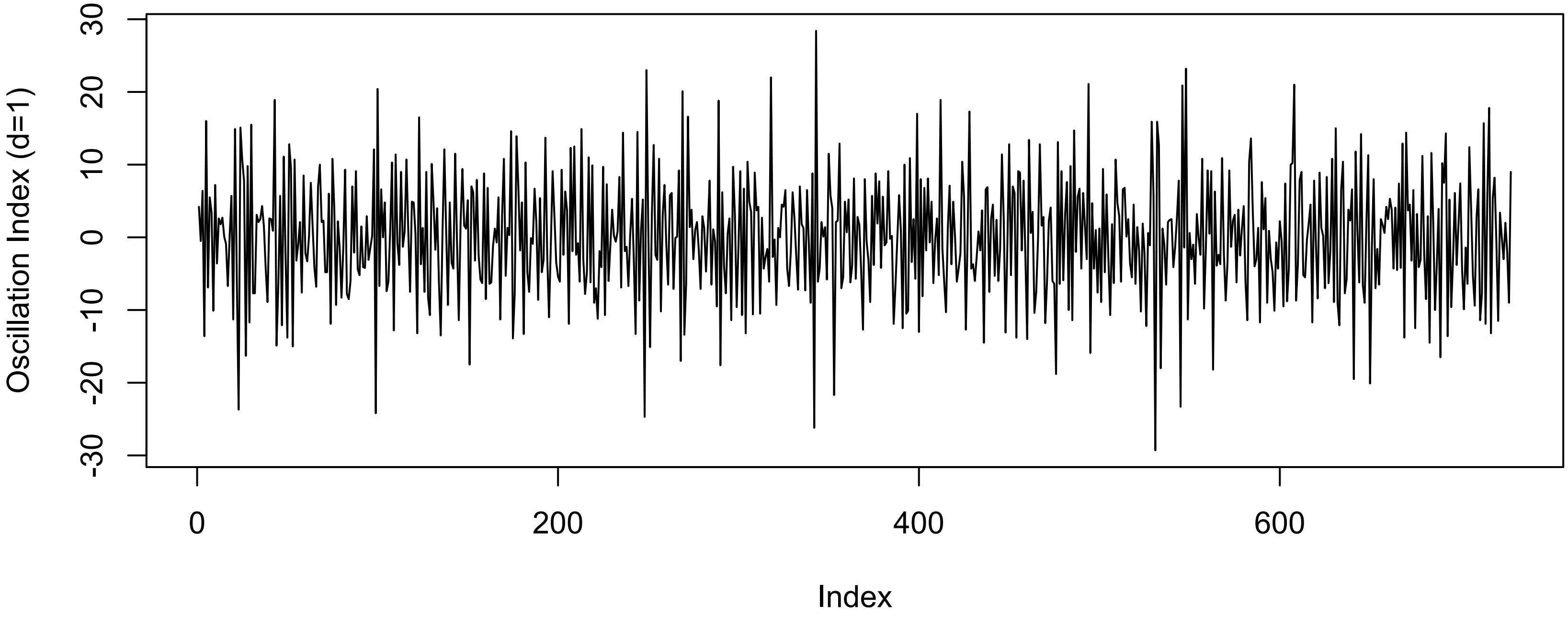

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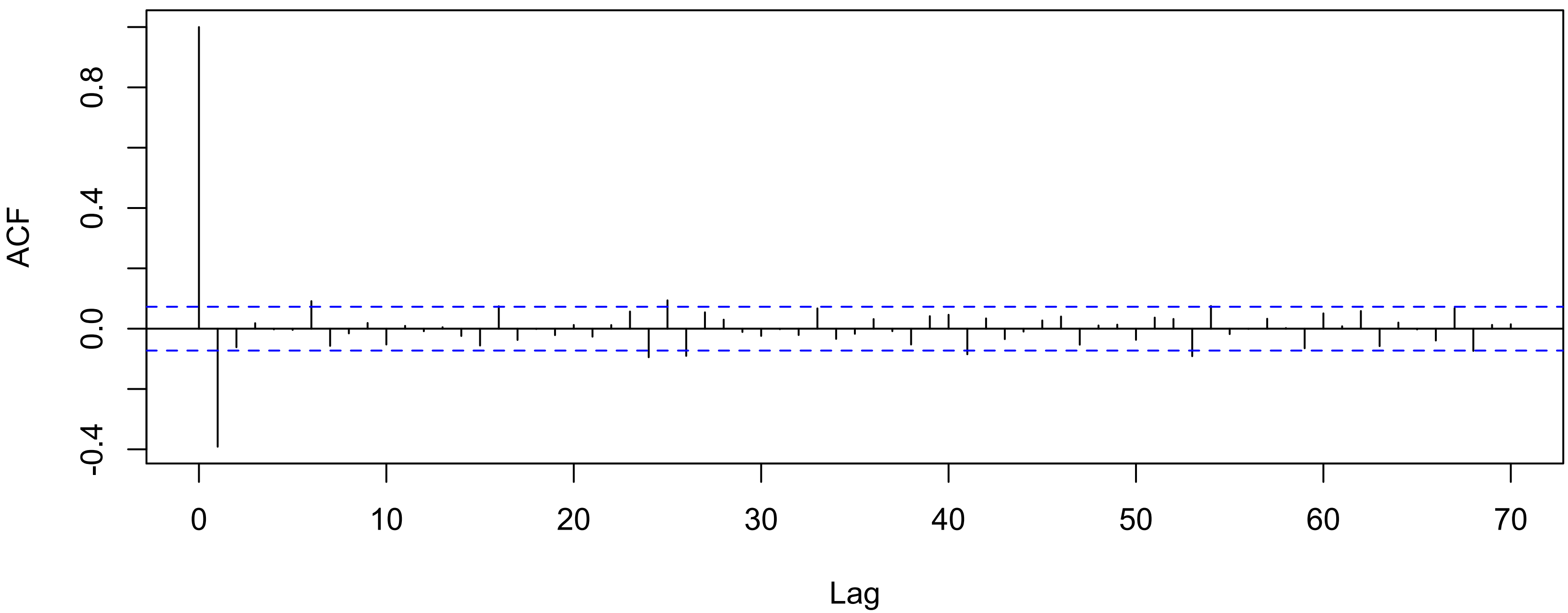

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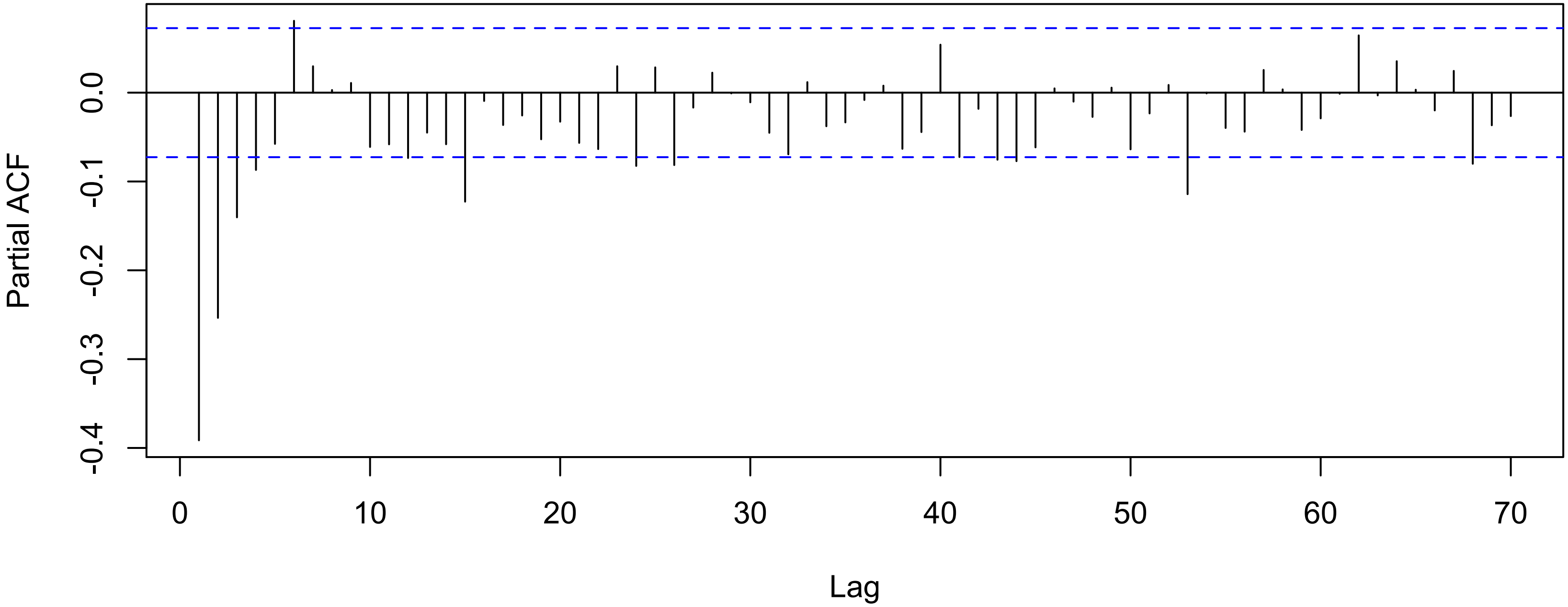

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Try ARIMA(0,1,1) model:

```
arima(x = x$Index, order = c(0, 1, 1))

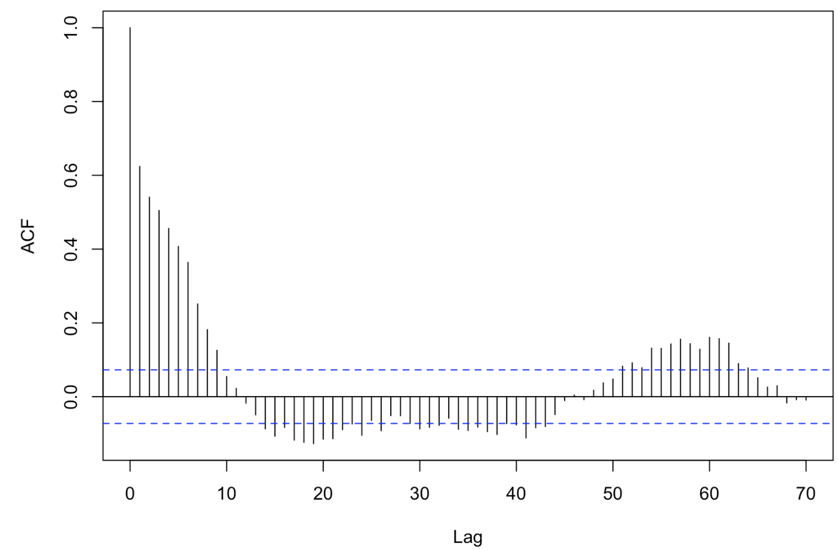
Coefficients:
          ma1
      -0.5579
s.e.   0.0308

sigma^2 estimated as 52.94:  log likelihood = -2477.98,  aic = 4959.96
```

Time series analysis

R functions:

acf(x,lag.max=70)



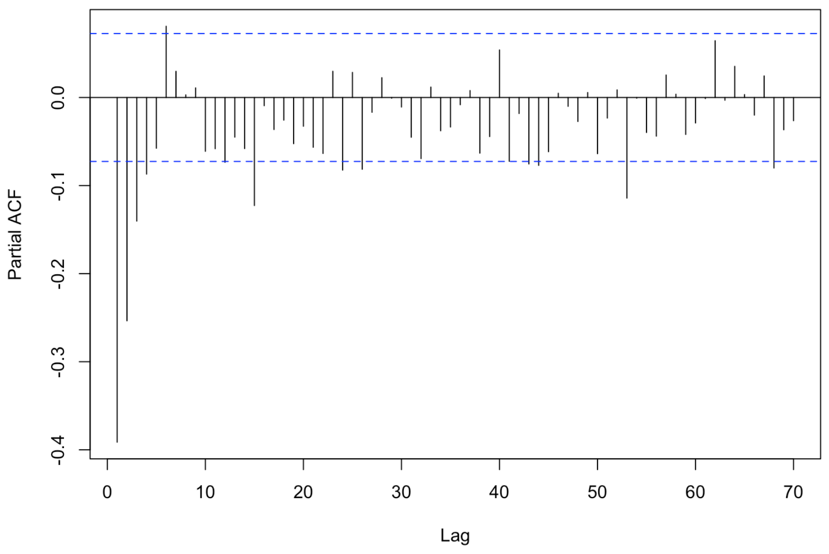
Time series analysis

R functions:

acf(x,lag.max=70)

diff(x)

pacf(diff(x),lag.max=70)



Time series analysis

R functions:

acf(x,lag.max=70)

diff(x)

pacf(diff(x),lag.max=70)

arima(x,order=c(0,1,1))

```
##
## Call:
## arima(x = x$Index, order = c(0, 1, 1))
##
## Coefficients:
##          ma1
##          -0.5579
## s.e.   0.0308
##
## sigma^2 estimated as 52.94:  log likelihood
= -2477.98,  aic = 4959.96
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