

(Source: O. Rueda, MRC-BSU; G. Marot, INRIA)

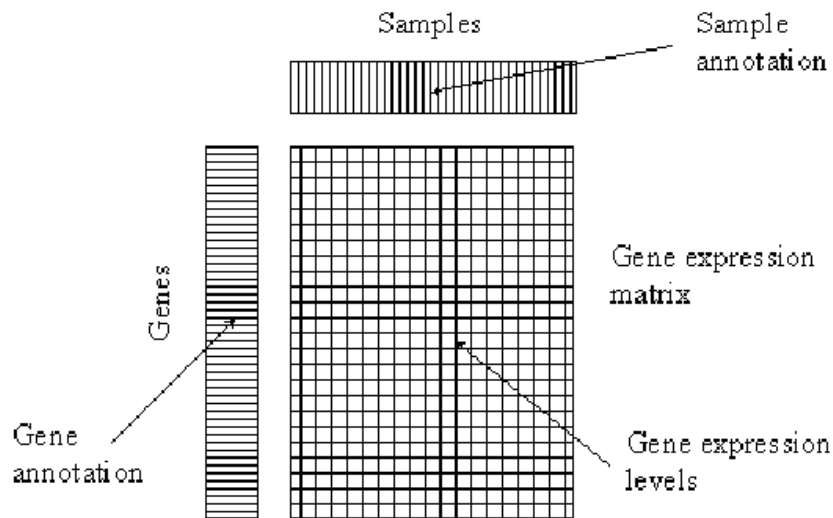
raw count for gene i, sample j

The mean is taken as "normalized counts" scaled by a normalization factor

one dispersion per gene

$$K_{ij} \sim \text{NB}(s_{ij}q_{ij}, \alpha_i)$$

Introduction



Introduction

```
> set.seed(777)
> cnts <- matrix(rnbinom(n=20000, mu=100, size=1/.25), ncol=20)
> cond <- factor(rep(1:2, each=10))

> dds <- DESeqDataSetFromMatrix(cnts, DataFrame(cond), ~ cond)
> dds <- DESeq(dds)
> results(dds)
```

log2 fold change (MLE): cond 2 vs 1

Wald test p-value: cond 2 vs 1

DataFrame with 1000 rows and 6 columns

	baseMean	log2FoldChange	lfcSE	stat	pvalue	padj
	<numeric>	<numeric>	<numeric>	<numeric>	<numeric>	<numeric>
1	97.3140	-0.682067	0.344525	-1.979730	0.0477339	0.745842
2	109.9860	-0.228819	0.450720	-0.507676	0.6116808	0.944354
3	98.8111	0.104291	0.462113	0.225683	0.8214483	0.978382
4	103.2615	0.306400	0.297682	1.029284	0.3033460	0.944354
5	97.9406	0.316338	0.357242	0.885501	0.3758864	0.944354
...
996	86.8057	0.0467703	0.287042	0.162939	0.8705668	0.980044
997	101.4437	-0.2070806	0.339886	-0.609264	0.5423495	0.944354
998	78.1356	-0.6372790	0.369515	-1.724637	0.0845930	0.824310
999	89.2920	0.7554725	0.306192	2.467314	0.0136131	0.614613
1000	103.5569	-0.0728875	0.348655	-0.209053	0.8344065	0.978382

Outline

▶ Part I: Quick recap

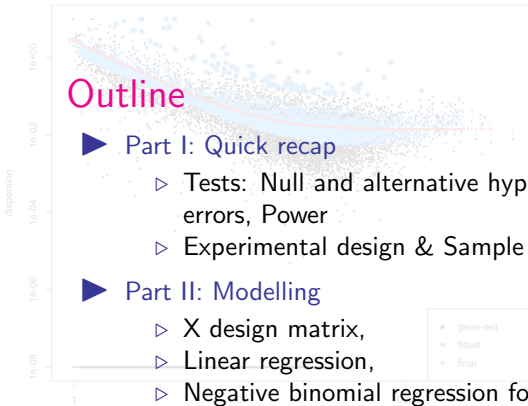
- ▷ Tests: Null and alternative hypotheses, Type I and type II errors, Power
- ▷ Experimental design & Sample size calculation.

▶ Part II: Modelling

- ▷ X design matrix,
- ▷ Linear regression,
- ▷ Negative binomial regression for counts.

▶ Part III: Multiplicity correction

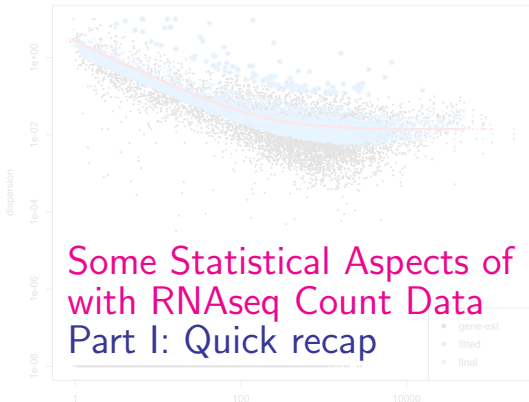
- ▷ Familywise error rate (FWER)
- ▷ False discovery rate (FDR)



The mean is taken as "normalized counts" scaled by a normalization factor

one dispersion per gene

$$K_{ij} \sim \text{NB}(s_{ij}q_{ij}, \alpha_i)$$



Some Statistical Aspects of DE Analysis with RNAseq Count Data Part I: Quick recap

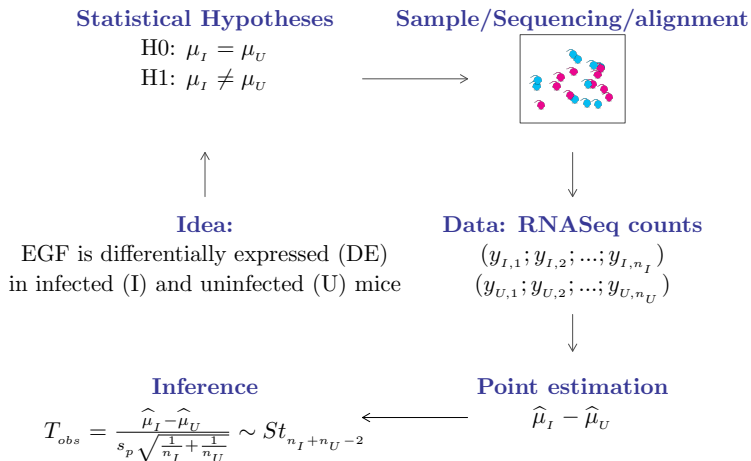
dominique-laurent.couturier@cruk.cam.ac.uk [Bioinformatics core]

The mean is taken as "normalized
counts" divided by a normalization
factor

one dispersion per gene

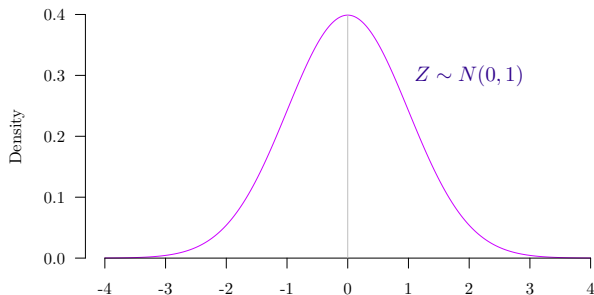
$$K_{ij} \sim \text{NB}(s_{ij}q_{ij}, \alpha_i)$$

Grand Picture of Statistics



Statistical tests

Assess how likely the observed test statistics is compared to the test statistics distribution under H_0 :



P-value for a two-sided test:

$$p\text{-value} = 2 \min [P(Z \leq Z_{obs} | H_0), P(Z \geq Z_{obs} | H_0)]$$

i.e. the probability of getting a test statistic as extreme or more extreme than the calculated test statistic if H_0 is true

Statistical tests

4 possible outcomes

Conclude:

- ▶ if $p\text{-value} > \alpha \rightarrow$ do not reject H_0 .
- ▶ if $p\text{-value} < \alpha \rightarrow$ reject H_0 in favour of H_1 .

		Test Outcome	
		H0 not rejected	H1 accepted
Unknown Truth	H0 true	$1 - \alpha$ [TN]	α [FP]
	H1 true	β [FN]	$1 - \beta$ [TP]

where

- ▶ α is the type I error, the probability of rejecting H_0 when H_0 is correct,
- ▶ β is the type II error, the probability of not rejecting H_0 when H_1 is correct.

Warnings

- ▶ 'absence of evidence is not evidence of absence',
- ▶ design may help minimising FP and FN (ie, maximising TN and TP).

Experimental design 1: Minimising biases

3 fundamental aspects of sounds experiments (Fisher 1935)

► Replication

Try to capture all sources of variability
(Biological versus technical variability)

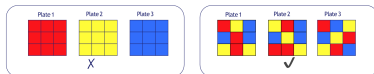
► Blocking

Try to remove technical biases/confounding
(Lane and batch effects)



► Randomisation

Try to remove confounding due to other factors



Experimental design 2: boosting power

Power- / Effect size- / Sample size- calculations

4 ingredients:

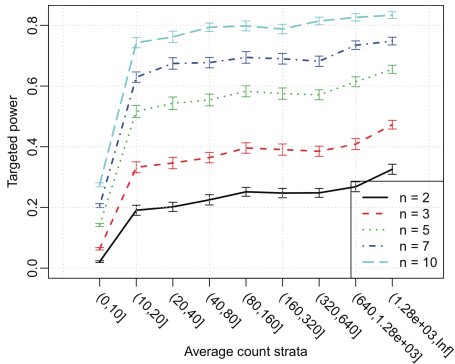
- ▶ $1 - \beta$, the power,
 - ▶ δ , the effect size: function of μ_U and μ_1
(log fold change, standardised difference),
 - ▶ n , the sample size (number of biological replicates),
 - ▶ α , the type I error.
- ▷ ϕ , nuisance parameters
(variability, sequencing depth, multiplicity correction)

'Give me 3 of them, I will deduce the fourth':

- ▶ **Power calculation:** Aim is to define the probability ($1 - \beta$) to detect an effect size of interest (δ) at the α level with a sample size of n biological replicates.
- ▶ **Sample size calculation:** Aim is to define the sample size (n) allowing to detect an effect size of interest (δ) at the α level with a given probability ($1 - \beta$).

Experimental design 2: boosting power

Power- calculations in DE analyses



(Wu, Wang and Wu (2015))

Coffee break



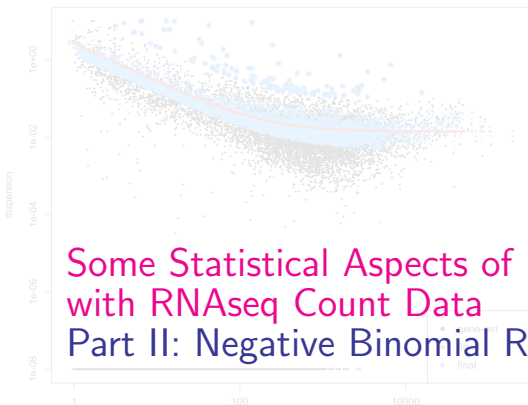


CANCER RESEARCH UK

CAMBRIDGE INSTITUTE



UNIVERSITY OF CAMBRIDGE



Some Statistical Aspects of DE Analysis with RNAseq Count Data

Part II: Negative Binomial Regression

dominique-laurent.couturier@cruc.cam.ac.uk [Bioinformatics core]

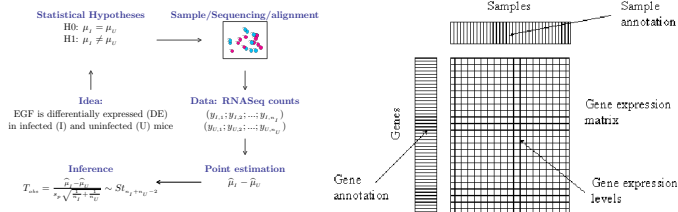
(Source: O. Rueda, MRC-BSU)

The mean is taken as "normalized count" multiplied by a normalization factor

one dispersion per gene

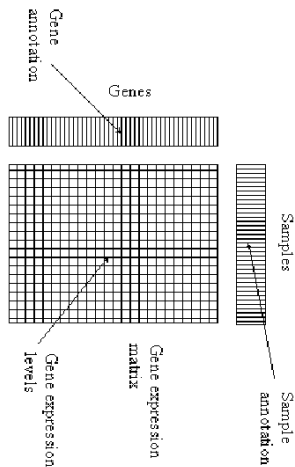
$$K_{ij} \sim \text{NB}(s_{ij}q_{ij}, \alpha_i)$$

Statistical modelling



Aim: Model the count data of each gene as a function of the conditions of interest (treatment, age, sex, batch, aso.)

Statistical modelling



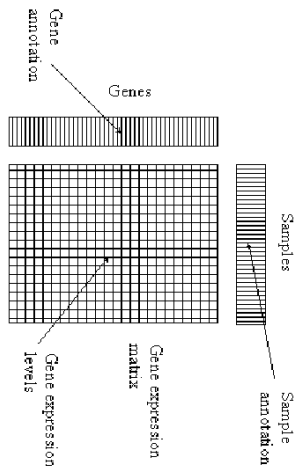
$$\mathbf{y} = f(\mathbf{X}) + \epsilon$$
$$E[\mathbf{y}] = f(\mathbf{X})$$

where

- ▶ \mathbf{y} denotes the $(n \times 1)$ vector of expression intensities of a given gene,
- ▶ \mathbf{X} denotes the $(n \times p)$ design/predictor matrix,
- ▶ ϵ denotes the $(n \times 1)$ stochastic error vector,
- ▶ $E[\mathbf{y}]$ denotes the expectation of \mathbf{y}

Express the count data vector of a given gene, \mathbf{y} , as a function f of characteristics of the samples (\mathbf{X} : age, treatment, aso) plus a stochastic error vector ϵ

Statistical modelling : Linear regression



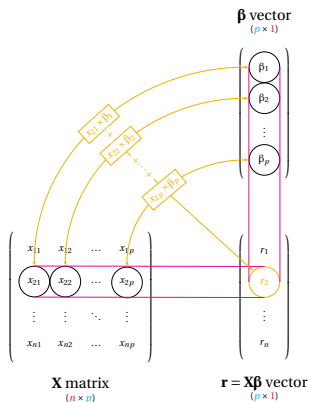
$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\epsilon}$$

$$\mathbb{E}[\mathbf{y}] = \mathbf{X}\boldsymbol{\beta}$$

where

- ▶ \mathbf{y} denotes the $(n \times 1)$ vector of expression intensities of a given gene,
- ▶ \mathbf{X} denotes the $(n \times p)$ design/predictor matrix,
- ▶ $\boldsymbol{\beta}$ denotes the $(p \times 1)$ parameter vector,
- ▶ $\boldsymbol{\epsilon} \sim N(0, \sigma^2)$ denotes the $(n \times 1)$ stochastic error vector,
- ▶ $\mathbb{E}[\mathbf{y}]$ denotes the expectation of \mathbf{y}

Statistical modelling : Linear regression



$$y = X\beta + \epsilon$$
$$E[y] = X\beta$$

where

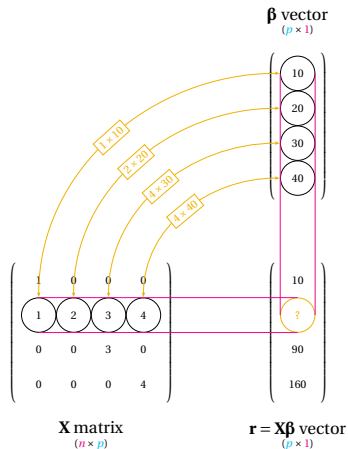
- ▶ y denotes the ($n \times 1$) vector of expression intensities of a given gene,
- ▶ X denotes the ($n \times p$) design/predictor matrix,
- ▶ β denotes the ($p \times 1$) parameter vector,
- ▶ $\epsilon \sim N(0, \sigma^2)$ denotes the ($n \times 1$) stochastic error vector,
- ▶ $E[y]$ denotes the expectation of y

Matrix multiplication:

the i th element $r = X\beta$ is obtained by

- ▶ multiplying **term-by-term** the entries of the i th row of X and each element of β ,
- ▶ and summing these products.

Statistical modelling : Linear regression



Matrix multiplication:

the i th element $r = \mathbf{X}\beta$ is obtained by

- ▶ multiplying **term-by-term** the entries of the i th row of \mathbf{X} and each element of β ,
- ▶ and summing these products.

Statistical modelling : Strategy

- ▶ Collect the information related to each sample for the predictors of interest,
- ▶ define β , the sets of parameters we are interested in,
- ▶ build the \mathbf{X} matrix that relates the sample information with the β
this step is automatically done in R by specifying the regression formula in the function `lm()` or `DEseq2()`
- ▶ estimate the β and use statistical inference to assess significance (p -values)
these two points are done by the function `lm()` or `DEseq2()`

Statistical modelling : $\mathbf{X}\beta$ (For information)

- ▶ Linear regression:

$$E[\mathbf{y}] = \mathbf{X}\beta,$$

- ▶ Cox regression:

$$h(t) = h_0(t)e^{\mathbf{X}\beta},$$

- ▶ Logistic regression:

$$\pi = \frac{e^{\mathbf{X}\beta}}{1+e^{\mathbf{X}\beta}},$$

- ▶ Mean expression levels for a given gene in DESeq2:

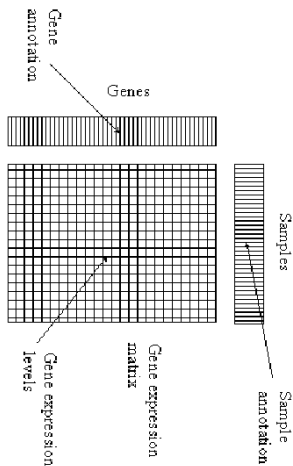
$$E[\mathbf{y}] = 2^{\mathbf{X}\beta},$$

Statistical modelling : X contrast matrix

We will discuss contrast matrices for models with

- ▶ 1 factor (1 categorical predictor),
 - ▷ 2 experimental conditions (binary predictor, like sex),
t-test
 - ▷ >2 experimental conditions,
One-way ANOVA
- ▶ 2 factors (2 categorical predictors),
 - ▷ without interaction,
 - ▷ with interaction,
Two-way ANOVA

Example: *Toxoplasma Gondii* Oocysts



#	Sample ID	Status	Time Point
1	SRR7657878	Infected	11 dpi
2	SRR7657881	Infected	11 dpi
3	SRR7657880	Infected	11 dpi
4	SRR7657874	Infected	33 dpi
5	SRR7657882	Uninfected	33 dpi
6	SRR7657872	Infected	33 dpi
7	SRR7657877	Uninfected	11 dpi
8	SRR7657876	Uninfected	11 dpi
9	SRR7657879	Uninfected	11 dpi
10	SRR7657883	Uninfected	33 dpi
11	SRR7657873	Infected	33 dpi
12	SRR7657875	Uninfected	33 dpi

2 Factors:

- ▶ **Status** with 2 levels (Infected/uninfected)
- ▶ **Time point** with 2 levels (11 dpi, 33 dpi)

Case 1: 1 two-level factor without intercept

Modelling 1:

- ▶ Mean expression level of gene 'G' is a function of Status: Uninfected and infected.
- ▶ 2 levels = 2 parameters

Sample information
(1 two-level factor)
I for 'Infected', U for 'Uninfected'

$$\begin{pmatrix} \mu_U \\ \mu_I \end{pmatrix} \quad \beta \text{ vector}$$

Parameters: $\beta = [\mu_U, \mu_I]^T$, where

- ▶ μ_U denoted the mean expression level for condition 'Uninfected'
- ▶ μ_I denoted the mean expression level for condition 'Infected'

I
I
I
I
U
I
U
U
U
U
I
U

$$\begin{pmatrix} \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \end{pmatrix}$$

X matrix
(11 × 2)

$$\begin{pmatrix} \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \end{pmatrix}$$

Xβ vector
(p × 1)

Case 2: 1 two-level factor with intercept

Modelling 2:

- ▶ Mean expression level of gene 'G' is a function of Status: Uninfected and infected.
- ▶ 2 levels = 2 parameters

Sample information
(1 two-level factor)
I for 'Infected', U for 'Uninfected'

$$\begin{pmatrix} \beta_0 \\ \beta_1 \end{pmatrix} \quad \beta \text{ vector}$$

Parameters: $\beta = [\beta_0, \beta_1]^T$, where

- ▶ $\beta_0 = \mu_U$ is the intercept and corresponds to the mean expression level for the reference group: condition 'Uninfected'.
- ▶ $\beta_1 = \mu_I - \mu_U$ is the difference in mean expression level between conditions 'Infected' and 'Uninfected'

I
I
I
I
U
I
U
U
U
U
I
U

$$\begin{pmatrix} \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \end{pmatrix}$$

X matrix
(11 × 2)

$$\begin{pmatrix} \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \end{pmatrix}$$

Xβ vector
(p × 1)

Design matrices for models with a two-level factor: R Code

Open the R Markdown Document ‘StatsRNAseq_Couturier.Rmd’
and go to Section ‘Contrast matrices / One 2-level factor’

```
> dds <- DESeqDataSetFromMatrix(cnts, DataFrame(cond), ~ cond)
```


Case 4: 1 three-level factor with intercept EXERCISE

Modelling 2:

- ▶ Mean expression level of gene 'G' is a function of Status: uninfected, half-infected and infected.
- ▶ 3 levels = 3 parameters

Parameters: $\beta = [\beta_0, \beta_1, \beta_2]^T$,
where

- ▶ $\beta_0 = \mu_U$ is the intercept and corresponds to the mean expression level for the reference group: condition 'Uninfected'.
- ▶ $\beta_1 = \mu_H - \mu_U$ is the difference in mean expression level between conditions 'Half-infected' and 'Uninfected'
- ▶ $\beta_2 = \mu_I - \mu_U$ is the difference in mean expression level between conditions 'Infected' and 'Uninfected'

Sample information
(1 three-level factor)
I for 'Infected', U for 'Uninfected'
H for 'Half-infected'

H
H
I
I
U
I
U
U
U
H
I
H

.	.	.
.	.	.
.	.	.
.	.	.
.	.	.
.	.	.
.	.	.
.	.	.
.	.	.
.	.	.
.	.	.
.	.	.

X matrix
(11 × 3)

β_0
 β_1
 β_2

β vector

.
.
.
.
.
.
.
.
.
.
.
.

Xβ vector
(p × 1)

Design matrices for models with a three-level factor: R Code

Open the R Markdown Document '[StatsRNAseq_Couturier.Rmd](#)'
and go to Section '[Contrast matrices / One 3-level factor](#)'

```
> dds <- DESeqDataSetFromMatrix(cnts, DataFrame(cond), ~ cond)
```

Case 5: 2 two-level factors without interaction

Modelling 1:

- ▶ Mean expression level of gene 'G' is a function of Status (Uninfected and infected) and Time (11 and 33 dpi).
- ▶ 2 (Status levels) \times 2 (Time levels) = 3 parameters without interaction

Parameters: $\beta = [\beta_0, \beta_1, \beta_2]^T$,
where

- ▶ $\beta_0 = \mu_{U,11}$ denoted the mean expression level for the reference group: condition 'Uninfected' at 'Time 11'
- ▶ β_1 denoted the shift in mean due to condition 'Infected'
- ▶ β_2 denoted the shift in mean due to condition 'Time 33'

Sample information
(2 two-level factors)
I for 'Infected', U for 'Uninfected'
11 for '11 dpi' and 33 for '33 dpi'

I 11
I 11
I 11
I 33
U 33
I 33
U 11
U 11
U 11
U 33
I 33
U 33

$\begin{pmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix}$

X matrix
(11 \times 3)

$\begin{pmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \end{pmatrix}$

β vector

$\begin{pmatrix} \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \end{pmatrix}$

X β vector
($p \times 1$)

Case 5: 2 two-level factors with interaction

Modelling 1:

- ▶ Mean expression level of gene 'G' is a function of Status (Uninfected and infected) and Time (11 and 33 dpi).
- ▶ 2 (Status levels) \times 2 (Time levels) = 4 parameters with interaction

Parameters: $\beta = [\beta_0, \beta_1, \beta_2, \beta_3]^T$,
where

- ▶ $\beta_0 = \mu_{U,11}$ denoted the mean expression level for the reference group: condition 'Uninfected' at 'Time 11'
- ▶ β_1 denoted the sift in mean due to condition 'Infected'
- ▶ β_2 denoted the sift in mean due to condition 'Time 33'
- ▶ β_3 denoted the sift in mean due to conditions 'Infected' & 'Time 33' jointly given the main effects of 'Status' and 'Time'

Sample information
(2 two-level factors)
I for 'Infected', U for 'Uninfected'
11 for '11 dpi' and 33 for '33 dpi'

I 11
I 11
I 11
I 33
U 33
I 33
U 11
U 11
U 11
U 33
I 33
U 33

$\begin{pmatrix} \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \end{pmatrix}$

X matrix
(11 \times 4)

$\begin{pmatrix} \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \end{pmatrix}$

X β vector
(p \times 1)

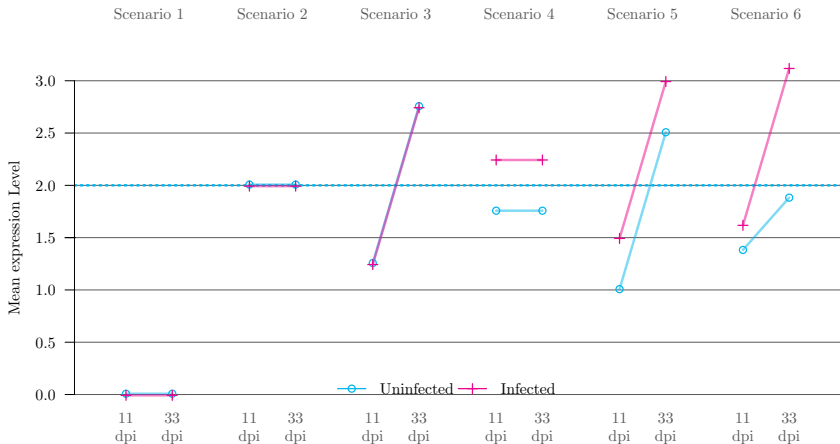
$\begin{pmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \\ \beta_3 \end{pmatrix}$

β vector

Models with 2 factors: possible scenarios

2 factors:

- ▶ Status (2 levels): Uninfected and infected
- ▶ Time (2 levels): 11 and 33 dpi



Design matrices for models with two two-level factors: R Code

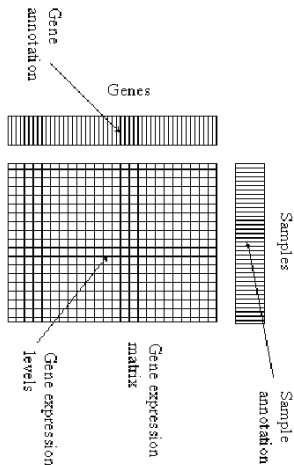
Open the R Markdown Document 'StatsRNAseq_Couturier.Rmd'
and go to Section 'Contrast matrices / Two 2-level factors'

```
> dds <- DESeqDataSetFromMatrix(cnts, DataFrame(cond), ~ cond)
```


Coffee break



Negative binomial regression: Model



$$y \sim \text{NB}(\mu, \phi)$$

$$E[y] = \mu = s 2^{\mathbf{X}\beta}$$

where

- ▶ y denotes the $(n \times 1)$ **count vector** of expression intensities of a given gene,
- ▶ \mathbf{X} denotes the $(n \times p)$ **design/predictor matrix**,
- ▶ β denotes the $(p \times 1)$ **parameter vector**,
- ▶ ϕ denotes the **dispersion parameter**,
- ▶ s denotes the **scaling factor vector** (library size),
- ▶ $E[y] = \mu$ denotes the expectation of y

Negative binomial regression:

Probability mass function

$$y \sim \text{NB}(\mu, \phi)$$

$$f(y|\mu, \phi) = \frac{\Gamma(y + \frac{1}{\phi})}{\Gamma(\frac{1}{\phi})\Gamma(y + 1)} \left(\frac{\phi\mu}{1 + \phi\mu} \right)^y \left(\frac{1}{1 + \phi\mu} \right)^{\frac{1}{\phi}}$$

with expectation and variance given by

► $E[y] = \mu = \mathbf{s} \mathbf{2}^{\mathbf{X}\beta}$

► $\text{Var}[y] = \mu \left(1 + \frac{\mu}{\phi} \right)$

Negative binomial regression: Log2 FC

```
log2 fold change (MLE): cond 2 vs 1
Wald test p-value: cond 2 vs 1
DataFrame with 1000 rows and 6 columns
```

	baseMean	log2FoldChange	lfcSE	stat	pvalue	padj
	<numeric>	<numeric>	<numeric>	<numeric>	<numeric>	<numeric>
1	97.3140	-0.682067	0.344525	-1.979730	0.0477339	0.745842
2	109.9860	-0.228819	0.450720	-0.507676	0.6116808	0.944354
...
999	89.2920	0.7554725	0.306192	2.467314	0.0136131	0.614613
1000	103.5569	-0.0728875	0.348655	-0.209053	0.8344065	0.978382

- ▶ $E[\mathbf{y} | \text{'cond 1'}] = 2^{\hat{\beta}_0}$
 - ▶ $E[\mathbf{y} | \text{'cond 2'}] = 2^{\hat{\beta}_0 + \hat{\beta}_1} = 2^{\hat{\beta}_0} 2^{\hat{\beta}_1}$
 - ▷ If not DE, $\beta_1 = 0$ so that $E[\mathbf{y} | \text{'cond 2'}] = 2^{\hat{\beta}_0} 2^0 = 2^{\hat{\beta}_0}$,
 - ▷ If DE, $\beta_1 \neq 0$ so that $E[\mathbf{y} | \text{'cond 2'}] = 2^{\hat{\beta}_0} 2^{\hat{\beta}_1}$
- Interpretation: *Multiplicative change in the mean gene expression level of $2^{\hat{\beta}_1} = 2^{-0.682067} = 0.6232717$ compared to the condition 1*

Negative binomial regression: Significance

```
log2 fold change (MLE): cond 2 vs 1
Wald test p-value: cond 2 vs 1
DataFrame with 1000 rows and 6 columns
```

	baseMean	log2FoldChange	lfcSE	stat	pvalue	padj
	<numeric>	<numeric>	<numeric>	<numeric>	<numeric>	<numeric>
1	97.3140	-0.682067	0.344525	-1.979730	0.0477339	0.745842
2	109.9860	-0.228819	0.450720	-0.507676	0.6116808	0.944354
...
999	89.2920	0.7554725	0.306192	2.467314	0.0136131	0.614613
1000	103.5569	-0.0728875	0.348655	-0.209053	0.8344065	0.978382

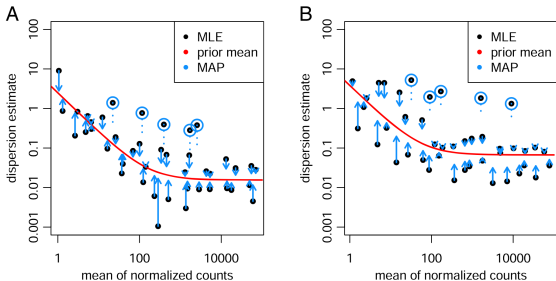
Wald Z-test to assess if a Log2 FC is significantly different from 0:

- ▶ **H0:** $\beta_1 = 0$ versus **H1:** $\beta_1 \neq 0$
- ▶ Z-statistic = $\frac{\hat{\beta}_1}{\hat{\sigma}_{\hat{\beta}_1}} = \frac{-0.682067}{0.344525} = -1.979730$
- ▶ P-value with $Z \sim N(0, 1)$ under **H0** is given by
> 2*(1-pnorm(abs(-1.979730)))

```
[1] 0.04773388
```

Negative binomial regression: Assumed Distribution

- ▶ The **assumed distribution of counts per condition for a given gene** depends on
 - ▷ $\hat{\beta}$, the estimate of the parameter vector,
 - ▷ $\hat{\phi}$, the estimate of the dispersion parameter for that gene.
- ▶ There are **3 ways to estimate ϕ in DESeq2**:
 - ▷ **gene-wise** dispersion estimates via ML (black dots) [not efficient],
 - ▷ **smooth curve** (red line) [strong assumption],
 - ▷ Bayesian **combination of both** [mid-way optimal solution].



Negative binomial regression: Assumed Distribution

```
-> mcols(dds)[,c("Intercept", "cond_2_vs_1", "dispGeneEst", "dispFit", "dispersion")]
```

```
DataFrame with 1000 rows and 5 columns
```

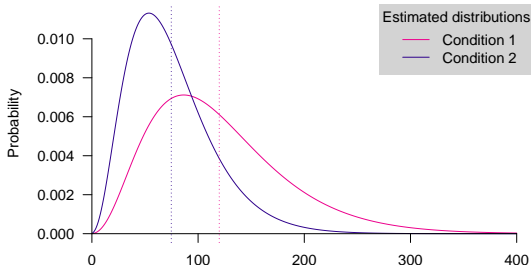
	Intercept	cond_2_vs_1	dispGeneEst	dispFit	dispersion
	<numeric>	<numeric>	<numeric>	<numeric>	<numeric>
1	6.90565	-0.682067	0.294082	0.234624	0.274708
2	6.89102	-0.228819	0.479231	0.230525	0.479231
...
999	6.05380	0.7554725	0.206644	0.229562	0.213730
1000	6.73029	-0.0728875	0.304930	0.235483	0.282745

- ▶ For gene 1 and condition 1, we have

$$y \sim \text{NB}(\hat{\mu} = 2^{6.90565} = 119.8969, \hat{\phi} = 0.274708)$$

- ▶ For gene 1 and condition 2, we have

$$y \sim \text{NB}(\hat{\mu} = 2^{6.90565} 2^{-0.682067} = 74.72831, \hat{\phi} = 0.274708)$$



Coffee break



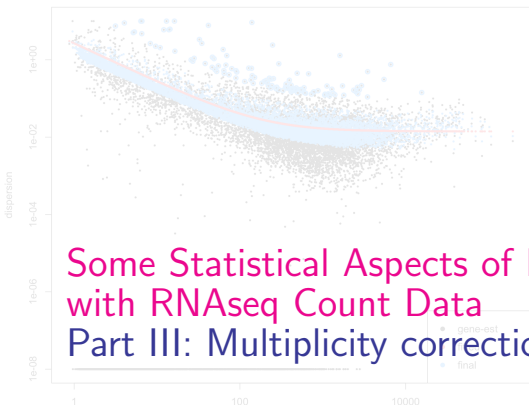


CANCER
RESEARCH
UK

CAMBRIDGE
INSTITUTE



UNIVERSITY OF
CAMBRIDGE



Some Statistical Aspects of DE Analysis with RNAseq Count Data Part III: Multiplicity correction

dominique-laurent.couturier@cruc.cam.ac.uk [Bioinformatics core]

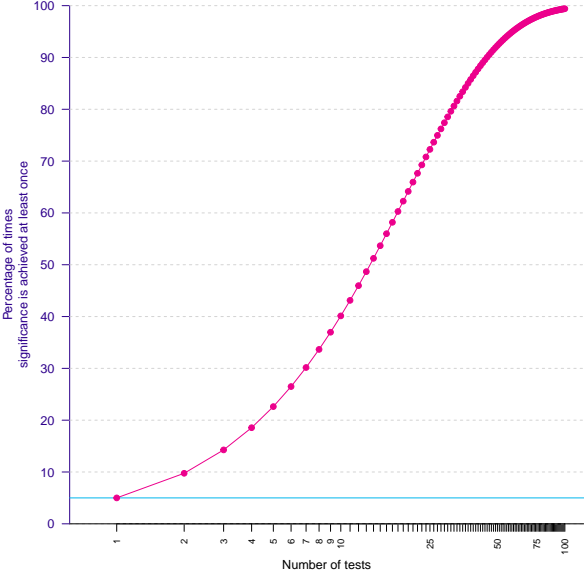
(Source: G. Marot, INRIA)

The mean is taken as "normalized
count" multiplied by a normalization
factor

one dispersion per gene

$$K_{ij} \sim \text{NB}(s_{ij}q_{ij}, \alpha_i)$$

Multiplicity correction: Familywise error rate



Multiplicity correction

Experimental design

Exploration

Normalization

Differential analysis

Multiple testing

The Family Wise Error Rate (FWER)

Definition

Probability of having at least one Type I error (false positive), of declaring DE at least one non DE gene.

$$FWER = \mathbb{P}(FP \leq 1)$$

The Bonferroni procedure

Either each test is realized at $\alpha = \alpha^*/G$ level
or use of adjusted pvalue $pBonf_i = \min(1, p_i * G)$ and $FWER \leq \alpha^*$.
For $G = 2000$, $\leq \alpha^* = 0.05$, $\alpha = 2.510^{-5}$.

Easy but conservative and not powerful.

Multiplicity correction

Experimental design

Exploration

Normalization

Differential analysis

Multiple testing

The False Discovery Rate (FDR)

Idea : Do not control the error rate but the proportion of error
⇒ less conservative than control of the FWER.

Definition

The false discovery rate of [Benjamini and Hochberg, 1995] is the expected proportion of Type I errors among the rejected hypotheses

$$\text{FDR} = \mathbb{E}(FP/P) \text{ if } P > 0 \text{ and } 0 \text{ if } P = 0$$

Prop

$$\text{FDR} \leq \text{FWER}$$

Multiplicity correction

```
> set.seed(777)
> cnts <- matrix(rnbinom(n=20000, mu=100, size=1/.25), ncol=20)
> cond <- factor(rep(1:2, each=10))

> dds <- DESeqDataSetFromMatrix(cnts, DataFrame(cond), ~ cond)
> dds <- DESeq(dds)
> results(dds)
```

log₂ fold change (MLE): cond 2 vs 1

Wald test p-value: cond 2 vs 1

DataFrame with 1000 rows and 6 columns

	baseMean	log2FoldChange	lfcSE	stat	pvalue	padj
	<numeric>	<numeric>	<numeric>	<numeric>	<numeric>	<numeric>
1	97.3140	-0.682067	0.344525	-1.979730	0.0477339	0.745842
2	109.9860	-0.228819	0.450720	-0.507676	0.6116808	0.944354
3	98.8111	0.104291	0.462113	0.225683	0.8214483	0.978382
4	103.2615	0.306400	0.297682	1.029284	0.3033460	0.944354
5	97.9406	0.316338	0.357242	0.885501	0.3758864	0.944354
...
996	86.8057	0.0467703	0.287042	0.162939	0.8705668	0.980044
997	101.4437	-0.2070806	0.339886	-0.609264	0.5423495	0.944354
998	78.1356	-0.6372790	0.369515	-1.724637	0.0845930	0.824310
999	89.2920	0.7554725	0.306192	2.467314	0.0136131	0.614613
1000	103.5569	-0.0728875	0.348655	-0.209053	0.8344065	0.978382

```
> p.adjust(results(dds)[, "pvalue"], method="BH")[c(1:5, 996:1000)]
```

```
[1] 0.7458417 0.9443538 0.9783822 0.9443538 0.9443538 0.9800445 0.9443538 0.8243099
[9] 0.6146133 0.9783822
```

Multiplicity correction

Experimental design

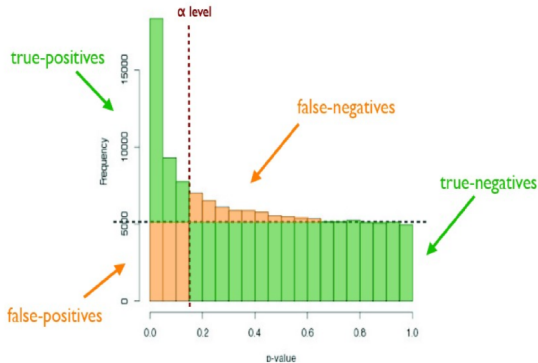
Exploration

Normalization

Differential analysis

Multiple testing

Standard assumption for p-value distribution



Source : M. Guedj, Pharnext

Multiplicity correction

Experimental design

Exploration

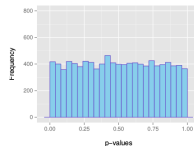
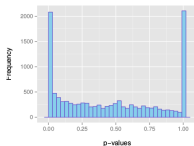
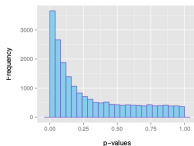
Normalization

Differential analysis

Multiple testing

p-values histograms for diagnosis

Examples of **expected overall distribution**



(a) : the most desirable shape

(b) : very low counts genes usually have large p-values

(c) : do not expect positive tests after correction

Multiplicity correction

Experimental design

Exploration

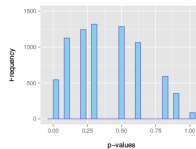
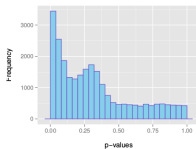
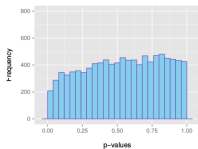
Normalization

Differential analysis

Multiple testing

p-values histograms for diagnosis

Examples of not expected overall distribution



- (a) : indicates a batch effect (confounding hidden variables)
- (b) : the test statistics may be inappropriate (due to strong correlation structure for instance)
- (c) : discrete distribution of p-values : unexpected

CONCLUSION

```
> set.seed(777)
> cnts <- matrix(rnbinom(n=20000, mu=100, size=1/.25), ncol=20)
> cond <- factor(rep(1:2, each=10))

> dds <- DESeqDataSetFromMatrix(cnts, DataFrame(cond), ~ cond)
> dds <- DESeq(dds)
> results(dds)
```

log2 fold change (MLE): cond 2 vs 1

Wald test p-value: cond 2 vs 1

DataFrame with 1000 rows and 6 columns

	baseMean	log2FoldChange	lfcSE	stat	pvalue	padj
	<numeric>	<numeric>	<numeric>	<numeric>	<numeric>	<numeric>
1	97.3140	-0.682067	0.344525	-1.979730	0.0477339	0.745842
2	109.9860	-0.228819	0.450720	-0.507676	0.6116808	0.944354
3	98.8111	0.104291	0.462113	0.225683	0.8214483	0.978382
4	103.2615	0.306400	0.297682	1.029284	0.3033460	0.944354
5	97.9406	0.316338	0.357242	0.885501	0.3758864	0.944354
...
996	86.8057	0.0467703	0.287042	0.162939	0.8705668	0.980044
997	101.4437	-0.2070806	0.339886	-0.609264	0.5423495	0.944354
998	78.1356	-0.6372790	0.369515	-1.724637	0.0845930	0.824310
999	89.2920	0.7554725	0.306192	2.467314	0.0136131	0.614613
1000	103.5569	-0.0728875	0.348655	-0.209053	0.8344065	0.978382